JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 33, No. 3, May–June 1996

Response of a Tactical Missile to Convected Aerodynamic Excitation

Mark S. Ewing*
University of Kansas, Lawrence, Kansas 66045

A tactical missile instrumented with pressure and acceleration sensors was flown on a fighter aircraft to help characterize the in-carriage flight environment. The pressure data associated with a throttle chop at a high angle of attack have been analyzed with two techniques to determine whether a simple, yet appropriate, model can be used to model forces applied to a structural finite element model. Analysis has revealed a nearly uniform convection of narrow-band random forces along the missile, indicating a highly coherent aerodynamic structure. The circumferential distribution of the standard deviation of measured pressures, which tend to swirl along the missile, seems to indicate the existence of shed-vortex-related swirl of the aerodynamic excitation. Lateral vibration of the missile has been analyzed using the finite element method, both with and without allowing for correlation (convection and swirl) within the aerodynamic excitation. In some frequency ranges these effects may be extremely important.

Nomenclature

c	= speed of sound in air
d	= distance between measurement locations
f	= frequency
$H_{ij}(f)$	= frequency response function relating a force at point i to the acceleration at point j
$R_{ij}(\tau)$	= cross-correlation function relating point i with point j
$S_k(f)$	= acceleration power spectral density at point k
$S_{ij}(f)$	= pressure cross-power spectral density relating measurements at points i and j
T	= period over which $R_{ij}(\tau)$ is calculated
t	= time
$x(t)$	= arbitrary function of time
$y(t)$	= arbitrary function of time
$\theta(f)$	= phase of $S_{ij}(f)$
τ	= time delay

Introduction

A TACTICAL missile often encounters a severe aerodynamic environment during external carriage on fighter aircraft. This environment is especially harsh for missiles located close to a rectangular engine inlet during a high-angle-of-attack maneuver with a throttle chop. In such a case, inlet corner vortices are known to spill out of the inlet (forming a horseshoe vortex). Concern over the ability to predict missile response in such conditions has spawned research on the prediction of the associated aerodynamics. Companion research efforts include the development of instrumented missiles to measure in-flight pressures[1] and efforts to determine the response of tactical missiles.[2]

In the late 1980s, an advanced medium-range air-to-air missile (AMRAAM) was modified to serve as a digital measurement vehicle (DMV) by personnel from the Structural Dynamics Branch, Structures Division of Wright Laboratory's Flight Dynamics Directorate (WL/FIBG, at that time). The DMV was fitted with accelerometers, pressure transducers, and a newly developed, miniature digital tape recorder. The vehicle was flown in a number of flight tests on a U.S. Air Force F-15 to determine whether the digital technology could be shown to be preferable to the analog system flown earlier.[1]

The pressure transducers were located at the stations and locations indicated in Fig. 1. The top of the missile is along the longitudinal line shown in Fig. 1b. The intent was to provide pressure measurements at three nearly equally spaced positions around the missile at the five selected longitudinal stations. However, the location of structures within the missile limited the available locations. The pressure transducers used were flush-mounted Kulites.

After the flight tests were flown, an analytical effort was begun to determine if the aerodynamic environment could be characterized to the extent that structural responses could be predicted from the measured pressure excitations.[2] The basis for measuring success was a direct comparison with the measured accelerations. Unfortunately, the level of success was not as expected, prompting the effort reported here. The major problem with the earlier study was that no clear physical model of the aerodynamics was available, and a somewhat crude method of applying loads was attempted. Another problem was that there was some uncertainty in the modeling of the elasticity of the connections between the missile and the launcher in the earlier study. An ongoing analysis takes into account the improved aerodynamic load modeling reported here as well as more realistic modeling of the missile support conditions.

Literature Review

Many investigators have considered methods to predict the pressure field surrounding a tactical missile in captive carriage. Similarly, many teams have actually measured pressures experienced in

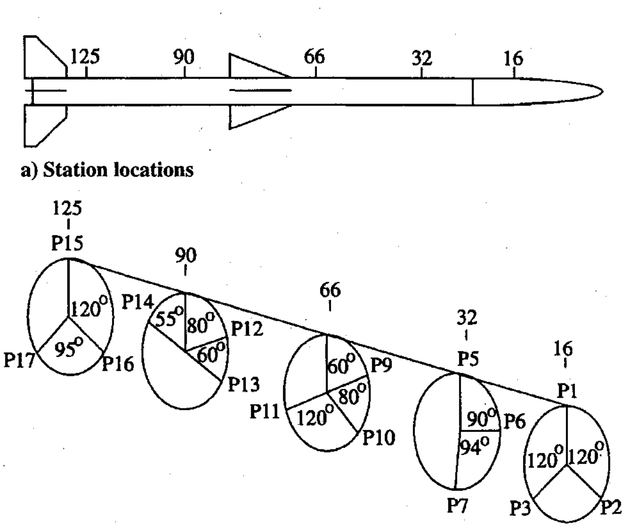


Fig. 1 Measurement locations on the DMV.

Received April 5, 1995; revision received Sept. 20, 1995; accepted for publication Dec. 15, 1995. Copyright © 1996 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

*Associate Professor, Aerospace Engineering, 2004 Learned Hall. Senior Member AIAA.

flight (e.g., Ref. 1). Further, the prediction of vibration response has been accomplished for many missiles. However, there does not appear to be any analysis in the open literature that includes correlation within the excitation pressures. On the other hand, such subtleties have been considered in other fields.

The classical problem of moving loads on structures has been reported by many authors, including Biggs.[3] Iwankiewicz and Sniady[4] studied the effect of random moving forces on a beam. Iwankiewicz[5] considered the more general problem of random forces moving at random velocities. More recently, Zerva[6] considered correlated ground motions on civil structures. The effect was important in some cases, in particular when the period of a structural mode is near in value to the fundamental period of the ground motion. Elishakoff[7] studied the effect of cross correlations of modes of vibration in a damped, modally dense shell. Once again, in some instances, the effects are important. In the field of acoustics, the problem of statistically correlated acoustical wavefronts has been studied for decades and is covered in many texts, including the excellent book by Bendat and Pearsol.[8]

Analysis of the Pressure Excitation

The AMRAAM DMV was flown for a number of flights on the F-15 LAU-106 ejector launcher, mounted at forward fuselage station number 7, on the right side of the aircraft. Data from numerous flight conditions were reviewed, with particular interest in throttle chops executed at high angles of attack. In such cases, inlet corner vortices are known to spill out of the inlets, forming a horseshoe vortex. An idealized vortex map at the plane of the engine inlet is depicted in Fig. 2. The DMV was attached to the F-15 at the location indicated, with the top of the missile oriented at 45 deg to vertical.

One flight condition has been analyzed in great detail because of the nearly 8 s of flight time during which the DMV appeared to be bathed in the flow spilling out of the engine inlet. The flight conditions recorded were as follows:

Altitude: 20,100 ft
Mach number: 0.89
Dynamic pressure: 537 psf
Angle of attack: 8.6 deg

The recorded AMRAAM DMV pressure data were analyzed using two techniques, one using the statistics of the amplitude of excitation and the other using the phase relationships between measurement locations.

Analysis Using Amplitude Statistics

A very simple way to look at the recorded data is to calculate statistical measures of the excitation across the missile. Without knowing the nature of the pressure field a priori, it may be true that the number of pressure transducers required to characterize the pressure field well enough for structural analysis is much larger than available on the DMV. Without a comparative measurement density study, however, there is no way to know. In this study, the approach is taken that one plausible way to characterize the excitation is to fit polynomial curves through measurement levels around the missile to see if any identifiable patterns emerge. Further, rather than fitting curves through pressures at instants in time (a most exhaustive pursuit), the curves were fitted through the means and standard deviations of the excitation pressures. The idea is that flow structures in the excitation would emerge from these measures if they were strong aerodynamic structures. It was expected at the outset that regions on the missile experiencing large standard deviations of excitation would be bathed by inlet vortices, while relatively quiet regions were not. The presence of such excitation segregation would indicate the existence of a steady vortex, such as the horseshoe vortex known often to spill out of a rectangular engine inlet.

Using an ensemble of 40,000 pressure measurements, recorded at 6400 Hz, 40 sets of 1000 records were used to compute the standard deviations and means at each of the 15 measurement locations. Then, a fourth-degree polynomial curve was fitted through the three computed values corresponding to the three measurement locations at each missile station. To provide the two additional data necessary to fit a fourth-degree curve, each of the fitted curves was required to have the same value and slope at 0 and 360 deg. The resulting set of five curve fits each for standard deviation and mean are measures of the distribution of pressure around the missile.

The pressure standard deviation and mean distributions for each of the five axial measurement stations along the missile are shown as carpet plots in Figs. 3 and 4. For each station, each of the 40 circumferential curve fits is plotted along the circumferential location axis. A carpet-plot surface was rendered in the figures by connecting the curve fits over the nearly 8 s during which data were recorded. Note that the distribution of the standard deviation seems to have more structure and much higher magnitude than that of the mean. Details of the data analysis, including raw data, are given in Ref. 9.

It was conjectured that an identifiable swirl would be apparent in these plots; for instance, a clear direction of rotation that is proportional to the distance along the missile was sought. However, the existence of a strong, coherent swirl is difficult to substantiate at this time. On the other hand, it is noted in Fig. 3 that the magnitude of the standard deviation at stations 32 and 66 is nearly twice that at stations 16 and 90. This disparity in magnitude indicates that aerodynamic disturbances are more pronounced at these locations. Further, a clockwise swirl (looking aft) may be indicated. This indication of swirl may be seen by looking at the Fig. 3 carpet plot for station 32, where the peak appears to stay fairly close to 200 deg from the top of the missile. In contrast, the peak for the station 66 plot appears to be at 100 deg. One conclusion is that a very coherent aerodynamic structure impinges on the missile near station 32; traverses aft, swirling 100 deg between stations 32 and 66; and dissipates or detaches prior to arrival at station 90. Although this conclusion cannot be completely justified by the data in this case, it is plausible and should be of concern to designers.

Analysis Using Phase Relationships

The knowledge that the AMRAAM often experiences severe wash from engine air dumping during a throttle chop suggests that significant aerodynamic disturbances are convected along the missile at a speed near the flight speed of the aircraft. Therefore, from the missile point of view, the narrow-band random excitation is very much temporally correlated. That is, a force or collection of forces arriving at a downstream point is just a time-delayed version of what an upstream point has already experienced. If the excitation also swirls, the convection can be along a helical rather than a lineal path. A straightforward way to identify a convected, or spatially correlated, excitation is to observe the temporal correlation function relating two pressures or the phase relationship in their cross power spectrum. The theory of this concept will now be presented.

Theory

When a single force moves at a constant velocity across the boundary of a structure, it will be recorded at discrete measurement locations with an arrival time varying in direct proportion to the distance

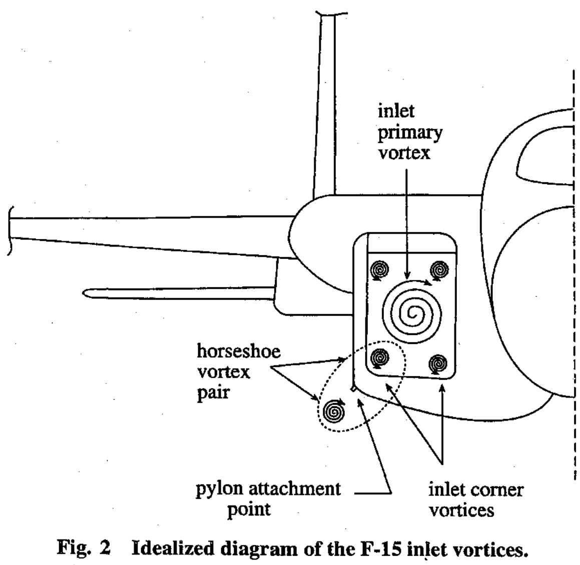


Fig. 2 Idealized diagram of the F-15 inlet vortices.

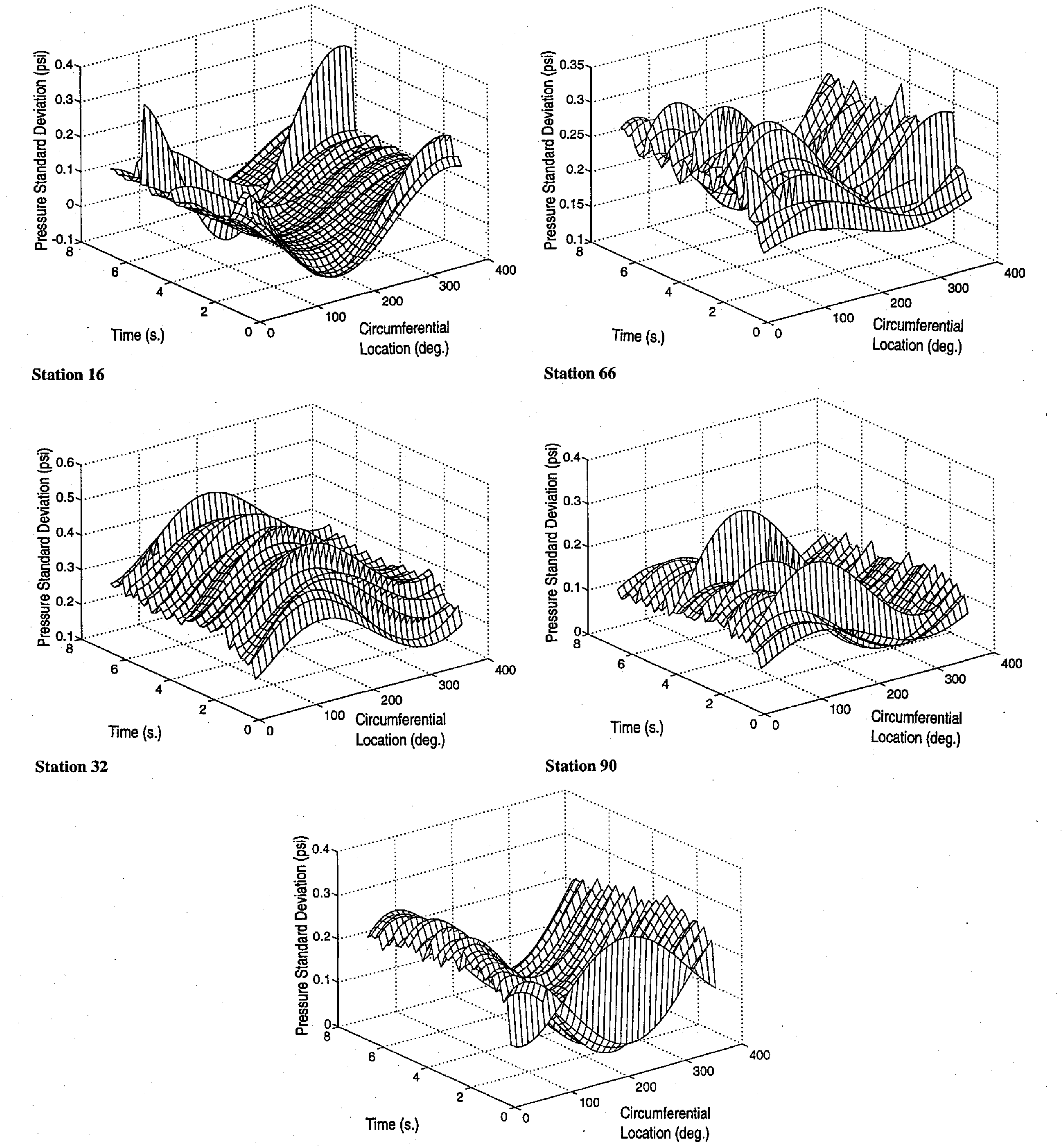


Fig. 3 Curve fits (around the DMV) of pressure standard deviations of 40 successive 1000-sample time slices.

between the points (and inversely proportional to the velocity). In other words, there is a delay, with respect to an upstream recording location, in the arrival time for a downstream recording location. The calculation of the temporal delay is very simply calculated as

$$\tau = d/c \tag{1}$$

The delay in arrival of an excitation at one point with respect to another results in a couple of interesting relationships. Consider first the cross-correlation function between two functions, $x(t)$ and $y(t)$, defined by

$$R_{ij}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)y(t+\tau)\,\mathrm{d}t \tag{2}$$

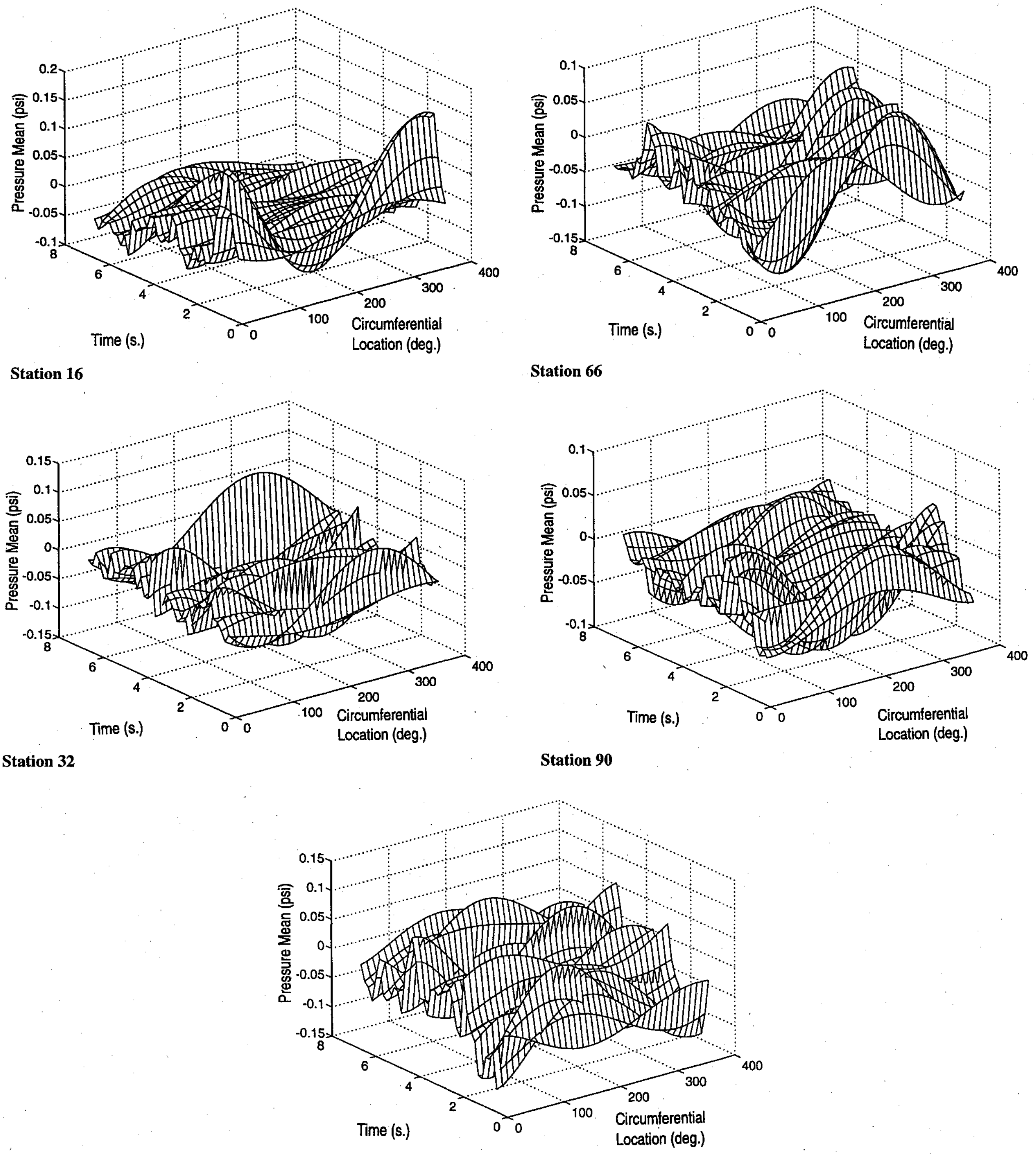
Now consider the cross-power spectral densities (PSD) of the two signals discussed above. The cross-PSD is

$$S_{ij}(f) = \int_{-\infty}^{\infty} R_{ij}(\tau)e^{-i2\pi f\tau}\,\mathrm{d}t \tag{3}$$

Finally, the phase angle in the cross-PSD in terms of a time delay is given by

$$\theta(f) = 2\pi f\tau \tag{4}$$

To summarize, then, the correlation between two measurement locations subject to a force moving past them both can be described in terms of a delay in the time-domain cross-correlation function or a phase change in the frequency-domain cross-PSD.



Station 16

Station 66

Station 32

Station 90

Station 125

Fig. 4 Curve fits (around the DMV) of pressure mean values of 40 successive 1000-sample time slices.

Analysis

Figure 5 shows the phase of the cross-PSD of three measurement pairs near the front of the DMV, namely, points 2, 6, and 9. Note the nearly linearly varying phase angle in the cross-PSD in at least the 200–300-Hz frequency range. This linear relationship is exactly what is expected according to theory for a coherent aerodynamic structure. Moreover, the slope of the phase change is generally in agreement with theory, i.e., the measurement locations farthest apart have the greatest rate of phase-angle change (because the time delay is greater). The analysis of these data is ongoing, with the hope that computational fluid dynamics (CFD) analysis will be able to shed light on the presence of vortex structures in the vicinity of the engine inlet.

Prediction of Missile Lateral Response

A NASTRAN model of the AMRAAM missile has been used to compute the lateral response of the missile to the measured pressure excitations. In an earlier study the aerodynamic excitations were characterized in terms of auto-PSD.[2] However, no correlation (i.e., no aerodynamic structure) in the excitation was considered. In this study, the excitation is modeled very simplistically to evaluate the importance of convection and swirl. Before describing the methods used, the theory of random analysis will be reviewed.

Theory

Once a linear structure has been represented in terms of its normal modes, one can predict the response to excitations that are

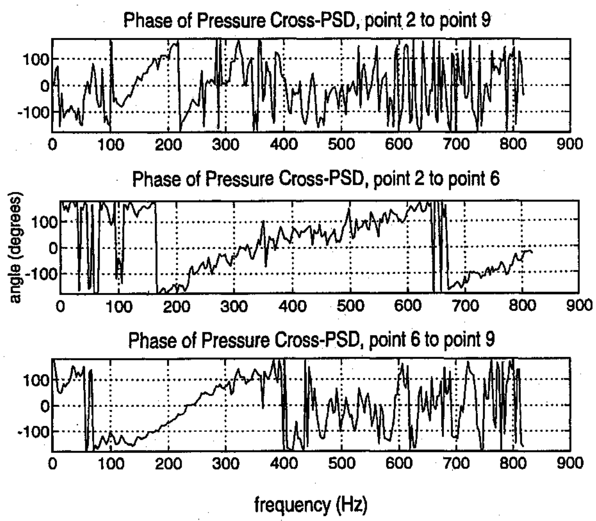


Fig. 5 Phase (in degrees) of the pressure cross-PSD relating measurement points 2, 6, and 9.

either deterministic or characterized by their statistical properties. The later approach is commonly taken for random excitations when the excitations are determined to be stationary, i.e., their statistical properties (mean, standard deviation, etc.) do not vary with time. The response to random excitation, is then predicted in terms of its statistics. The relationship[8] between stationary random excitation and acceleration response is

$$S_k = H^*_{ki} S_{ij} H_{jk} \tag{5}$$

Note that in the case where $i = j$, S_{ij} refers to the auto-PSD of the excitation.

In the case in which the excitations are uncorrelated, i.e., there is no statistical relationship between the excitations, all cross-PSD are zero, and the response relationship is

$$S_k = |H_{ki}|^2 S_{ii} \tag{6}$$

Such an uncorrelated response relationship is the one used in Ref. 2 and most other structural response analyses. However, the use of correlated response modeling in the acoustics community is fairly common.

Note that S_{ij} in Eq. (5) is a complex function, which, in practice, is only estimated as a discrete function by virtue of digital Fourier transform calculations performed on an ensemble of time-history data. Further, the calculations typically involve averaging over large numbers of data sets. The result is either a vector of complex numbers or a vector of amplitudes and phase angles. Either combination represents the correlation relationships in the physical process. Noise and nonstationary characteristics of the process being measured often degrade the estimate of the process statistics, especially with regard to phase information. This degradation often obscures correlation, which is evident only if there is a noticeable phase relationship. Therefore, uncorrelated excitation analysis is the norm, and correlated excitations are often unobserved and, even more often, simply not calculated.

Analysis Methodology

The NASTRAN structural analysis code has the capability to solve for the vibration response power spectra, as described in Eqs. (5) and (6). In an earlier study,[2] the response was calculated for uncorrelated excitations constructed from measured pressure data using Eq. (6). The agreement of analysis and test was not completely acceptable, as responses were often in disagreement by multiplicative factors of 2 or 3. As shown previously, however, there is ample evidence of convection at a speed near the forward speed of the aircraft in flight test. Therefore, an analysis has been completed to quantify the effect of including this convection, using Eq. (5).

In the case of a set of random forces moving at a uniform speed, or convected, along a beam, the time delay τ [Eq. (1)] can easily be encoded within either the frequency response function or the cross-power spectra. Encoding the time delay in the frequency response function is particularly simple with NASTRAN, and so this method was used in the correlated excitation analyses reported.

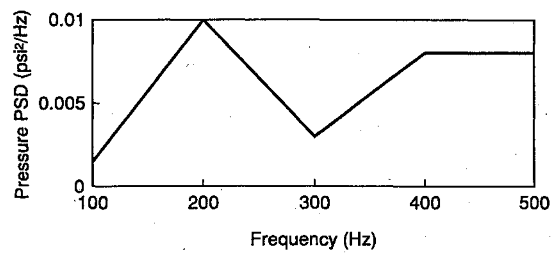


Fig. 6 Pressure PSD of the random excitation applied to a beamlike model of the AMRAAM missile.

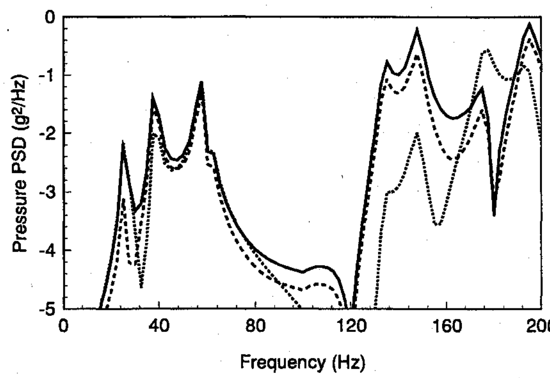


Fig. 7 Predicted lateral vibration of a beamlike structure because of random excitation: . . . . ., uncorrelated; ——, convection; and - - - -, convection with swirl.

To clearly show the effect of correlations in the excitation, simulated aerodynamic excitation has been used. By using simulated excitation, the effect of swirl can be clearly discerned, as the response is not affected by other processes, such as aircraft motion. Flight-test data show that this effect can dominate the response in some cases, but typically provides the driving force for approximately half of the response of the real AMRAAM missile. Therefore, for a complete analysis, both aerodynamic forces and aircraft motion must be considered.

Based on pressures measured for the DMV on the F-15, a generic random process was formulated and is characterized by the pressure PSD presented in Fig. 6. This generic PSD was based on the PSD from six measurement locations near the front of the missile. The PSD is characterized by a narrow-band, peaked process centered at 200 Hz and a flat-top process above 400 Hz. These random forces were applied to the finite element model for the AMRAAM in three cases. In the first, no correlation was allowed. Secondly, the forces were modeled to be convected along the missile at a speed of 11,000 in./s. This rate is approximately the speed of the flight-test aircraft when the pressures used to formulate the pressure spectrum were recorded. The convection form of correlation was invoked by specifying a time delay for force arrival that is proportional to the distance along the missile model. Thirdly, swirling convection was added to the model by specifying circumferential time delays corresponding to a uniform swirl along the missile. The total swirl angle along the missile was 90 deg.

Results

Figure 7 shows the effect of including the temporal correlation in the aerodynamic excitation. The lateral displacement of the missile (in the plane through the top of the missile) is shown for the cases in which the excitation is 1) uncorrelated, 2) correlated because of uniform longitudinal convection, and 3) correlated because of uniform longitudinal convection and a total swirl angle of 90 deg. Note that the analysis including the correlations identified in the aerodynamic analysis predicts substantially higher response in the 120–160-Hz range. This frequency range is important because it includes the fundamental frequency for many electronic boards in tactical missiles. However, note that including swirl in the modeling has the effect of reducing the lateral response of the missile in three low-frequency modes between 20 and 60 Hz.

The results presented are for idealized, convected aerodynamic excitation in the absence of aircraft motion. Therefore, one must consider the magnitude of aerodynamic excitation relative to base motion to determine whether allowing for correlation in the excitation is important. Also, note that in this analysis no attempt was made to allow for the effect of swirl on fins and wings. These appendages were included in the finite element model, but no attempt was made to apply swirling aerodynamic loads to them. Clearly, however, swirling aerodynamic excitation would have a much greater effect on the amplitude of lateral vibration if the interaction with fins and wings were considered.

Conclusions

The presence of temporal correlation in the flow past a missile near an engine inlet has been identified by analysis of measured pressures on a missile. The analysis reveals strong convection, as expected, and at least some evidence of swirl. The demonstration was based on both cross-power spectral phase-shift analysis (for the convection) and on statistical analysis of curve-fitted measurements (for the swirl).

The importance of properly characterizing aerodynamic excitation in a structural analysis has also been demonstrated. Convection of a strong aerodynamic structure along a missile markedly increases the lateral vibration levels in some frequency intervals. In the cases studied, this increase occurs in a frequency range that can excite sensitive electronic components within the missile.

Cooperative research with CFD analysts is clearly indicated if a better view of flow near an engine inlet is to be realized. Also, measurement systems with a more dense pressure transducer network are necessary. In this study, the pressure field was characterized circumferentially with a fourth-order polynomial, on account of the small number of data available. The resulting model of the pressure field allowed only a very basic view of the aerodynamic structures present. However, recent advances in the miniaturization of flight-quality measurement systems make many more pressure measurements on a missile entirely feasible.

Acknowledgments

The author would like to thank the following people: Dick Talmadge, formerly of U.S. Air Force Wright Laboratory, for having the vision to create the DMV; Larry Huttsell, of U.S. Air Force Wright Laboratory, for help with gaining access to the AMRAAM data used in this study; Dansen Brown, of U.S. Air Force Wright Laboratory, for help with analyzing the data, both during this study and in many earlier studies; and Farshid Haste, a former graduate research assistant, for help with the aerodynamic data analysis.

References

[1]Anon., "AMRAAM Instrument Measurement Vehicle (IMV) Captive Flight Test Report," Hughes Aircraft Co., Rept. 32050-2963, CDRL A03S, Contract F08635-82-C-0001, Ref. No. E76091, MSG 51226, Feb. 1986.

[2]Anon., "AMRAAM Model User's Guide," Deliverable 3 from GSA task "Model Prediction of AMRAAM Vibration Levels," Dayton Operations of Atlantic Research Corp., GSA CR GS00K88AFD2587, GSA Task Order CCC571625, June 1991.

[3]Biggs, J. M., *Introduction to Structural Dynamics*, McGraw–Hill, New York, 1964, pp. 315–328.

[4]Iwankiewicz, R., and Sniady, P., "Vibration of a Beam Under a Random Stream of Moving Forces," *Journal of Structural Mechanics*, Vol. 12, No. 1, 1984, pp. 13–26.

[5]Iwankiewicz, R., "Response of Vibratory Systems to General Random Pulses," *Journal of Sound and Vibration*, Vol. 108, No. 3, 1986, pp. 429–446.

[6]Zerva, A., "Pipeline Response to Directionally and Spatially Correlated Seismic Ground Motions," *Journal of Pressure Vessel Technology*, Vol. 115, Feb. 1993, pp. 53–58.

[7]Elishakoff, I., "On the Role of Cross-Correlations in the Random Vibrations of Shells," *Journal of Sound and Vibration*, Vol. 50, No. 2, 1977, pp. 239–252.

[8]Bendat, J. S., and Pearsol, A. G., *Engineering Applications of Correlation and Spectral Analysis*, Wiley, New York, 1980, pp. 188–197.

[9]Ewing, M. S., and Haste, F., "Analysis and Characterization of Measured Aerodynamic Loading on a Tactical Missile," U.S. Air Force Wright Lab., TR 94-3156, Feb. 1995.

R. M. Cummings
Associate Editor